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Assessment of the construction process of the Cathedral of Tortosa

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Abstract

The development of new measurement techniques is changing the process of documentation and assessment of built heritage. An accurate planimetric basis is essential for the constructive evaluation of a building. This paper presents the methodology used in the geomètric assessment of Gothic vaults in the Cathedral of Tortosa (Spain). The use of a Terrestrial Laser Scanner allows the assessment of the vaulting to a level of detail that has never been possible to achieve previously. The accurate identification of geometric deviations in the masonry can be combined with the information from primary sources, revealing new information about the construction process of the apse.

Keywords

Vaults; Gothic; Cathedral of Tortosa; construction process; terrestrial laser scanning

Introduction

The construction process of historical buildings is a key issue in the preservation of heritage. Moreover, in disciplines such as the history of art, the precise chronology is fundamental. Current methodologies are based on historiographical and archaeological techniques, where the assessment of primary sources provides reliable, but sometimes incomplete, information. In these cases it is necessary to complement data with other procedures. Furthermore, an accurate surveying of the geometry allows the identification of displacements and possible damages to the structure, which are major subjects in architectonical diagnosis.

In the case of historical constructions, such as cathedrals, the geometrical survey of the masonry presents two main difficulties: the complexity of the architecture and the inaccessibility of the ceilings. Traditional techniques require a great amount of time and resources, but the appearance of new technologies is changing the situation. The use of massive data capture techniques for architectural surveying, such as the Terrestrial Laser-Scanner (TLS) and Digital Photogrammetry, has become widespread over the last decade.[1] They enable the geometrical properties of objects to be recorded within a short time and at a level of accuracy that was previously very difficult to achieve. Thus, it is possible to obtain a three-dimensional digital model of an object that can be assessed through computerised procedures. Nowadays a significant number of software packages can be found that can be used for this purpose, obtaining data such as Cartesian coordinates, bi-dimensional sections, parametric 3D models, etc.

The methodology used in the geometric analysis of the Gothic apse of Tortosa Cathedral, built between 1383 and 1441,[2] is set out below. The use of a TLS has made it possible to obtain a highly accurate three-dimensional model, in which it is possible to parameterise geometrical deviations in the masonry in detail, identifying Cartesian coordinates of strategic points in the 3D model. The treatment of the data enables the identification of formal differences between the several vaults, which can be related with the construction process of the apse.

The results of the methodology enable a comparative study from the point of view of diferent disciplines, using primary sources from this period, such as the Llibres d'Obra (Ll.o., Cathedral construction accounts), which are preserved in the Archivo Capitular de Tortosa (ACTo., Chapter Archive of Tortosa). Thus, the available historical documentation can be ascertained and completed with the geometrical data that previously were inaccessible.

The tradition of the geometrical section in Gothic architecture

The theoretical framework of the Gothic project determines the geometric reference parameters in the survey of the Cathedral of Tortosa. The assessment of the section of a medieval masonry construction is complex, since medieval builders were bound by the professional secret of the geometria fabrorum.3 The Status of Saint-Michele of Strasbourg (1563) forbade revealing the layout of the section, built through the essential points of the plan by means of squares (Strasbourg 1563, art.13)4.

The combined knowledge of ecclesiastical promoter and medieval builder set the geomètric proportions of Gothic construction. This relationship was highlighted by Wilhelm Worringer (1881–1965) and Erwin Panofsky (1892–1968).5 The search for the principles] of the canons of medieval creation is partly found in the cosmology of the Timaeus of Plato (c. 429–347 BC), as is recognised by Francis Macdonald Cornford (1874–1943).6 Thus, one of the contact points between clerical and artisan statements is set in the measure and proportion of architecture.

From the point of view of Gothic cathedrals, Otto von Simson (1912–1993)7 approached the question of cult resources, looking for evidence in the Civitatis Deis, De Ordine and the Musica of Saint Agustín (354–420). This research was supplemented by examining authors such as Boecio (480–524) and his De consolatione philosophiae and the Musica, and the main commentators on Plato, such as Calcidius, Marciano Capella or Macrobius.8

The renewed search for creative canons of Platonic geometry of the Timaeus has been developed by Carmen Bonell,9 and continued by Nigel Hiscock,10 based on the references of the Stromatesis by Clemente de Alejandría (c. 150–c. 215), the Hexaëmeron by Basilio el Grande (c. 330–379) and the De hominis opifico by Gregorio de Nisa (c. 335–c. 395). These works set the relationship between primary sources and the knowledge of clerical promoters of Gothic cathedrals. Otherwise, Robert Bork11 emphasises the geometrical origin of creative necessity in Gothic design. These considerations take as reference several studies regarding French cathedrals, among others the work of Fernie,12Kidson13 and Murray.14

The construction of the Cathedral of Milan (1386) and the discussions about its construction (1392 and 1401) is a primary source that allows an epistemological approach to determine the methodology of these geometric and arithmetic developments. The Master Builders of Milan discussed the use of proportions ad triangulum and ad quadratum from French models, or other lower proportions.15 The C. Cesarino Vitruvian edition illustrated the section of Cathedral of Milan16 (Figure 1) and explained a method to obtain the height of the main vaults in ratio with the width of the lateral chapels and the central nave.

The proportional Gothic theory derived from this medieval tradition has direct application. The ad quadratum section has a proportion of 1/1, while the ad triangulum section is based on the relationship between the height and side of the equilateral triangle. During the Gothic period, the height of an equilateral triangle had to be calculated by the approach of the square root of the number three. Gerberto of Aurillac (997–999) proposes a solution for this immeasurable number through the proportion 7/6.17 On the other hand, Gabriele Stormalocho (1391) uses the proportion of 8/7 in the Cathedral of Milan.18

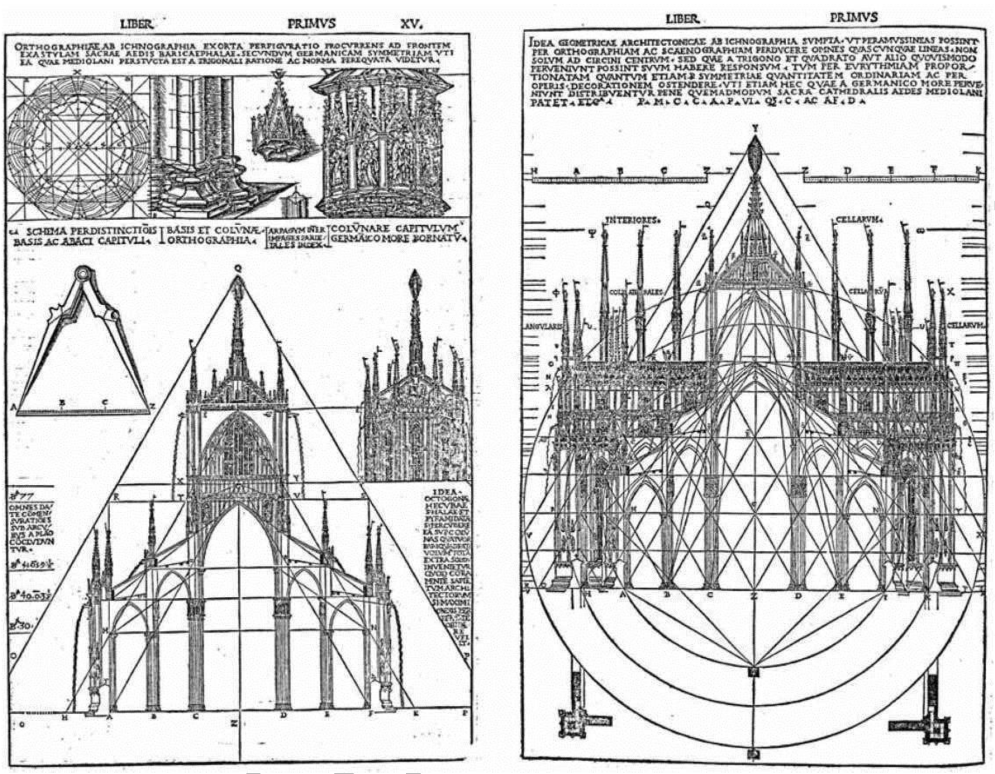

Figure 1. Cathedral of Milan, layout of the section by Cesare Cesarino (Vitruvio, 1521)

Studies of Gothic buildings indicate the application of ad quadratum and ad triangulum proportion.19 Other smaller proportions, such as Pitagoric or Neoplatonic with harmonic modulation, have been suggested.20 This is the framework of the theoretical section in Gothic construction, where a direct relation of proportion is found between the points in the plan and its section. The theoretical section of Tortosa will be determined following these principles, and it will be the reference to detect and assess topographical deviations in the new geometrical survey of the masonry vaults.

The Gothic apse of Tortosa Cathedral

Theoretical model

The theoretical framework that regulates the typological arrangement of Tortosa Cathedral can be found in the main historical studies of Gothic construction.21 The layout of the Gothic project is governed by proportional rules that establish the main measures of the construction. Thus, the section has a modular relation linked with the floor plan measure. A detailed review of the theoretical framework of Gothic design related to the Cathedral of Tortosa and its layout can be found in Lluis.22

In typological terms, the vaults of the heptagonal apse of Tortosa are located at three diferent heights. Also, the radial chapels are connected visually, with the traditional separating wall being replaced by a pillar on the ambulatory. The elimination of the wall between the chapels had been tentatively attempted at Santa Maria de la Aurora in Manresa (1328), with a cross-section ad triangulum of 8/7. This wall was completely eliminated in Tortosa around 1377, establishing a lower cross-section ratio of 9/5. This structural issue appeared years later in the debate over the Cathedral of Milan (1392).23

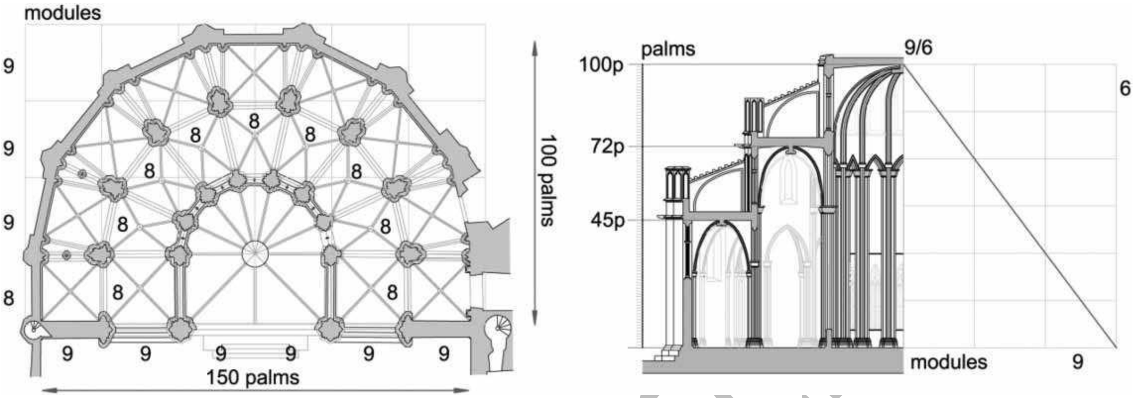

Figure 2. Metrology of the apse of Tortosa Cathedral

At the beginning of the project, the Master established the unit of measurement for the new cathedral (1347).24 The basic standard for the measurements found in the Ll.o. (ATCo) is the cana, 25 which is equivalent to 8 palms, the palm being equivalent to 12 fingers. A comparison of the documents standardising the Tortosa cana with the one used in Barcelona (24-VII-1593) shows that the Tortosa cana used in the cathedral measures 1,858 cm and the palm measures 23.23 cm.26

An exhaustive assessment of the metrological proportions determined the main dimensions of the apse:27 150 palms in width, 100 palms in depth and height. The layout of the floor plan can be defined through the measures of radial chapels. These are square and have interior measurements of 21×21 palms. The interior main axis of the pillars of the apse, where the work was laid out, is equidistant at 24 palms, three Tortosa canas. They were stacked out at 54 palms from the centre of the presbytery. There is a ratio between the radius of the circumference (54 palms) 18 modules, and the side of the polygon of 14 sides (24 palms) 8 modules, establishing a ratio of 9/8 (Figure 2).

The theoretical deployment model of the vaults has a ratio of 9/5 in the radial chapels, and 9/6 in the rest of the heading. This means that the keystone had to be 10.45 m (45 palms) high in the chapels, 16.73 m (72 palms) in the ambulatory and 23.23 m (100 palms) in the presbytery.

Chronology

The apse with double ambulatory of the Gothic Cathedral of Tortosa was built between 1383 and 1441. The chronology of the construction process is partially recorded in the Llibres d'Obra (Ll.o, cathedral construction accounts), preserved in the Chapter

Archive in Tortosa. This primary source contains the accounting of several years, and has been analysed in depth by Almuni.28 It has been possible to determine the construction chronology with precision by means of the church construction accounts (Ll.o.) (Figure 3).

The construction of the new cathedral began in 1347, but the work on the Gothic Building came to a standstill shortly after, due to the Black Death (1347) and the War of the two Peters (1356–1369). Construction resumed in the late fourteenth century with the works on the heptagonal apse.

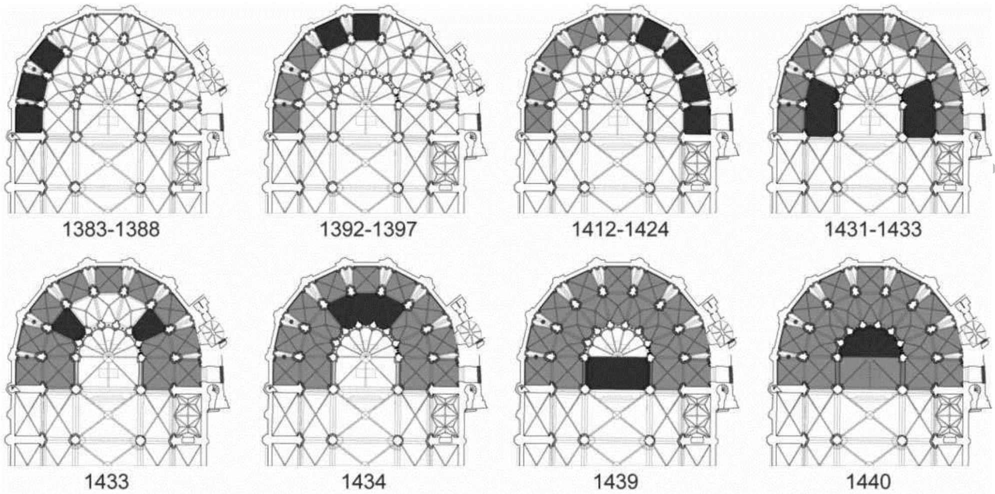

Figure 3. Constructive development of the radial chapels and the ambulatory (J. Lluis, 2002)

In the initial phase of construction a ring of nine radial chapels was built around the Romanesque cathedral, which remained in use. They were built sequentially, from the Gospel side (North) to the Epistle (South) side, between 1383 and 1424, and have a square floor plan, with ribbed vaults. The theoretical cross-section of the keystones was topped at 45 palms (1045.35 cm) high.29

The second phase saw the construction of nine vaulted ceilings in the ambulatory (1424–1435). Two are square, and seven are trapezoidal, with a theoretical deployment of the crosssection of the keystone at 72 palms (1672.56 cm) high. Unlike the radial chapels, the vaults were built symmetrically, from the mouth of the apse towards the choir, in a West–East direction.

Finally, the roof of the presbytery (1435–1441) was topped with a keystone of 10 palms of diameter (232.3 cm), which had to be placed exactly 100 palms (2323 cm) high.30 According to the measurements of the floor plan, the cross-section of the radial chapels has a ratio of 9/5 (9 units high by 5 at the base) while in the ambulatory and in the presbytery it has a ratio of 9/6.

Construction and geometric variations

It is possible to establish the precise sequence of the construction process of the vaults. There are welldocumented examples, such as the trapezoidal sector of the ambulatory, located opposite the Chapel of San Vicente. There, the scaffolding was placed on 24 July 1433, it was shored up on 31 July, the vault was built between 1 and

5 August and it was taken down on 27 August . This final operation completed the structural process, which took 35 days. The filling of this vault was built between 2 September and 13 September, as well as the inner wall and the tiling of the outer finish. This was done in two layers, and this operation completed the masonry work. A ribbed vault in the ambulatory was built and covered in just eight weeks.31 Geometrical displacements during construction may be produced by different causes: (1) constructive techniques and processes, and/or (2) settling of the vaults after removing the formwork.

The first case depends on geometric aspects, the height of the scaffolding and the tension of the vaults. It also depends on the execution of the work and the setting of the mortar joints. Finally, the formwork may suffer deformations or translations.

In the second case, the settling can occur just after the removal of the formwork or after a long period of time. Following removal of the formwork, the masonry arch begins to push against the buttresses, which inevitably yield slightly. Thus, the span increases by a small amount, and the geometry of the arch is recomposed to accommodate the movement.32 This may sometimes lead to a descent of the keystone. Over time, there may also be some settling in the masonry, with a possible impact on the geometrical conditions of the structure.

Geometrical survey

The first computerised topography of Tortosa Cathedral was carried out between 1995 and 2000, for the Sancta Maria Dertosae Master Plan.33 The data capture for the survey was conducted using direct measurements, referenced to polygonal points fixed by a total station. Numerical data was stored using CAD applications, providing a survey with a margin of error of approximately 3 cm.

The methodology used provides a planimetry of the floor plan with sufficient accuracy, but a detailed survey of the vaults would require a major investment of resources. In this case therefore, measurements of only some of the vaults were taken, based on singular points.

In 2013, a new survey of the cathedral was made using a Terrestrial Laser scanner.34 The machine used was a C10 Leica laser scanner,35 a compact device for mid-range measurements which uses the time-of-flight principle, with a complete field of view (360° horizontal, 270° vertical) and an accuracy of 6 mm per point up to a range of 50 m. Images are captured at each station in order to specify the colour of the points.

The complexity of the architectural layout required 32 stations to avoid occlusions in the model, and 25 artificial targets, strategically distributed to reference each position within the same coordinate system. Using a medium density scanning mesh a cloud of over 457 million points was obtained (Figure 4).

The points in the cloud were processed using the application Cyclone (Leica Geosystems),36 which places the clouds of points in the same local coordinate system. Then they are processed with the program 3D Reshaper37 to obtain the mesh. The 3D model is generated with a TIN mesh (Triangulated Irregular Network) and an average triangle size of 2.5 cm.

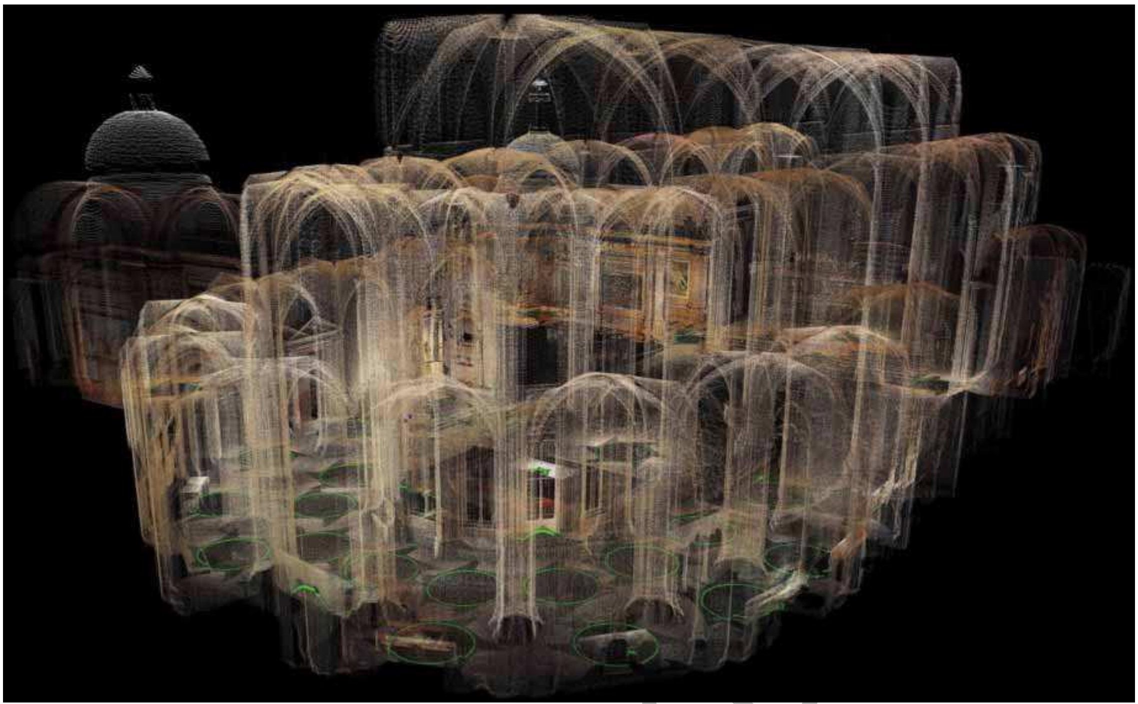

Figure 4. Point Cloud of the Cathedral of Tortosa (A. Costa and J. M. Puche, 2013).

Assessment of the 3D model

After processing the point cloud and generating the 3D mesh, we have a scale model within a known system of (x, y, z) coordinates. The model allows highly accurate direct measurements of the geometry, and cross-section lines can be obtained directly from the mesh. Thus, it is possible to parameterise the coordinates that define the geometry of the vaults.

The strategy used involves working from cross-section plans to accurately identify the desired points. Therefore, for a detailed study of the vaults, pairs of sections were produced using axial coordinates, according to the inflections of the severy (Figure 5). CAD applications are used in this step, and the coordinates obtained are entered into an Excel spreadsheet for the treatment of the data. To avoid errors in the measurement of heights due to irregularities in the ground, the average ground level in the first chapel of the Gospel side is set at the elevation benchmark 0.

The assessment is focused on the heights of the points which defines the geometry of the severies and the keystones.

A nomenclature is established for each vault and for the significant points of their geometry (Figure 6). As regards the geometric points, a distinction is made between main sections of the vaults (SI and SII).

Having determined the characteristic points, numerical values are listed to undertake a comparative study of the deviations in each vault. Although there are apparently no significant differences between those of the same type, their topography is not uniform and it has been possible to quantify the deviations.

Results of the geometrical assessment

Ratio of heights – Chapels

The height of the keystones (Ha) in the chapels (Figure 7) differs by up to 40 cm, with the keystone located at the greatest height in C2 (10.253 m) and at the lowest height in C8 (9.851 m). The next two chapels are of a similar height (C4 and C5) with a difference that does not reach 1 cm. The last four tend to be lower. The height of the severy in the keystone (Hb) follows the same pattern as that described for the keystone, with small variations and an average distance from Ha of 41 cm.

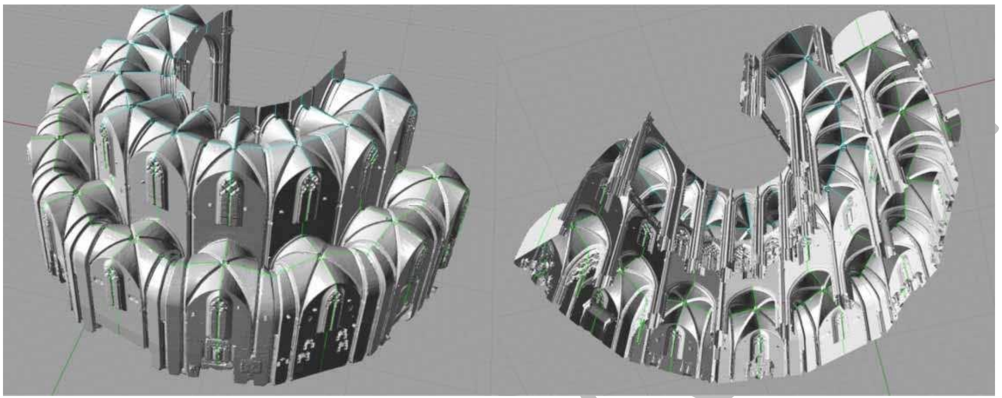

Figure 5. Sections of the 3D model (A. Costa and J. M. Puche, 2013)

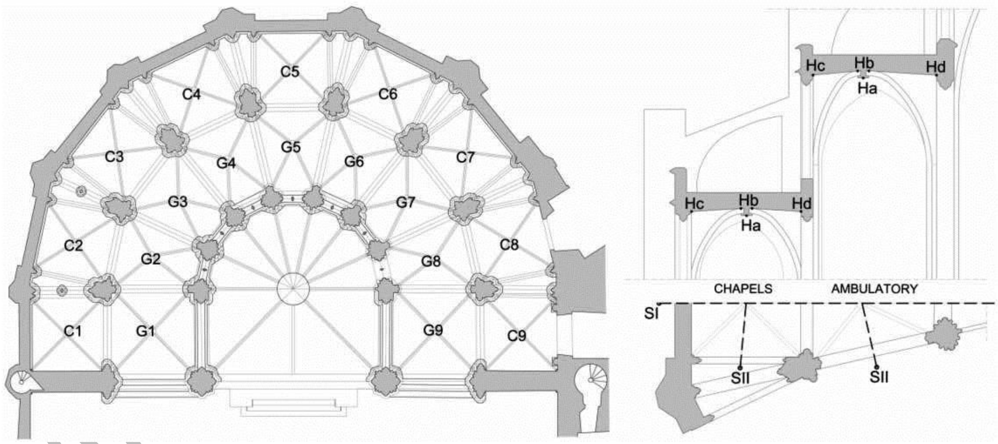

Figure 6. Nomenclature criteria

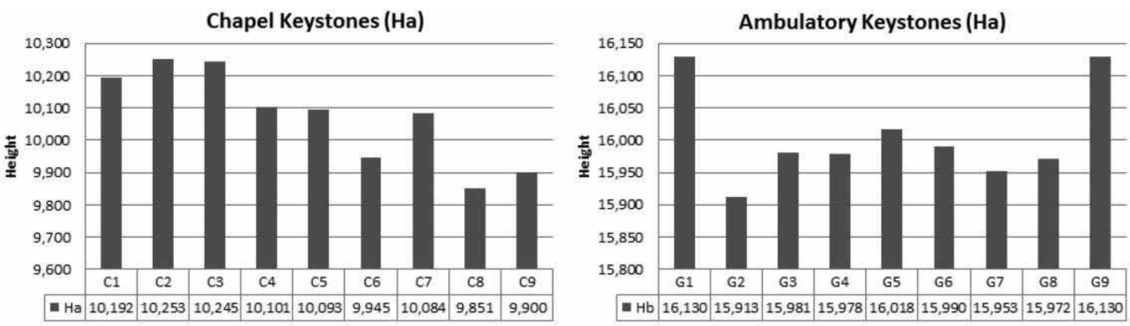

Figure 7. Heights of the keystones

If we compare the heights of the severy where it meets the former arch (Hd) and the wall (Hc) in the longitudinal section (SI), Hc is always located higher than Hd. The dispersion in the former is 17.5 cm (C1–C4) and in the wall it is 15.10 cm (C3–C7). The height of the first three vaults is lower than the others (Figure 8).

Values reach a greater range in the case of radial sections (SII), and the relation of heights between both sides of the severy changes (Hc>Hd is not always true). Likewise, the first three chapels also tend to be lower, with ranges between 17.9 cm for Hc (C2–C9) and 13.6 cm for Hd (C1–C8).

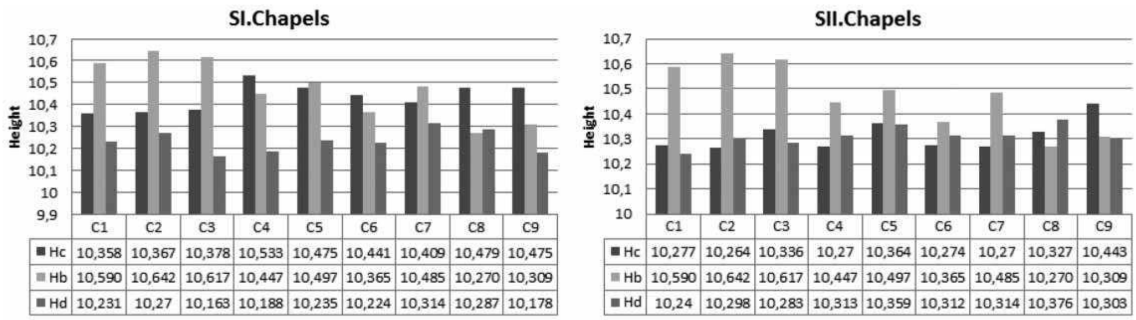

Figure 8. Heights of the severies in the chapels. SI and SII.

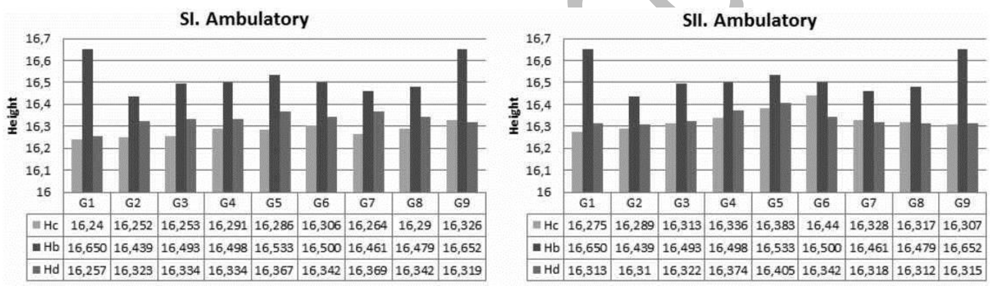

Figure 9. Heights of the severies in the ambulatory. SI and SII.

Ratio of heights: ambulatory

The height of the keystones in the vaults (Ha) (Figure 7) of the ambulatory follows a diferent pattern. Their height ranges from 21.8 cm (G1–G2), with the vaults placed in the mouth of the apse reaching the greater heights, together with the central one. The height tends to decrease, and is regained at the centre. As in the chapels, the height of the severies in the keystone (Hb) follows the same pattern, with a difference of height from Ha of about 50 cm.

For Hc and Hd in S.I. (Figure 9), except for G8, Hc is lower than Hd, in contrast to the vaults of the chapels. The dispersion in the former is 8.6 cm (G1–G9) and in the second it is 11.2 cm (G1–G7). In the case of S.II the heights range 16.5 cm for Hc (G1–G6) and 9.5 cm for Hd (G2–G5).

Discussion

Comparative analysis based on the available sources and the results obtained from the threedimensional model allows to relate the masonry executed and the historical chronology of construction.

The construction process

The participation of several Masters in the construction of the radial chapels could be determined through the sources38 and some adjustments identified in the carvings and mouldings of the masonry. After the initial Mastership of Mestre Joan (1378–c. 1385), three main periods of construction of the radial chapels are identified, with the successive Masterships of Pere Moragues (c. 1382–1387), Joan de Maini (c. 1382–1403/14010) and Pasqual Xulbi (c. 1402/1410–c. 1416/1420), who was succeeded by his son, Joan Xulbi (c. 1416–1458). The first period (1377–1383), involved the construction of chapels C1, C2 and C3 (San Pedro, San Pablo and San Vicente). Afterwards, between 1387 and 1397, came the construction of the next two chapels. The third period (1412–1424) saw the construction of the remaining four chapels. The construction was successive from the chapel of the gospel (C1) to the chapel of the epistle (C9).

Based on an analysis of the relationship obtained between the geometric points, there is a clear correspondence between the succession of phases of construction and the geometric variations. The keystone of the first three vaults (C1 to C3) is higher than the others, and the severies follow a pattern clearly different from the rest. C4 breaks with the previous pattern, and there is a successive stabilisation of the pattern, with the keystone at a lower height and similar relationships between the other points. The pattern changes significantly once again in the last two chapels (C8 and C9), which could suggest a change in the mastership, which is not dated in the LL.o (Figure 10).

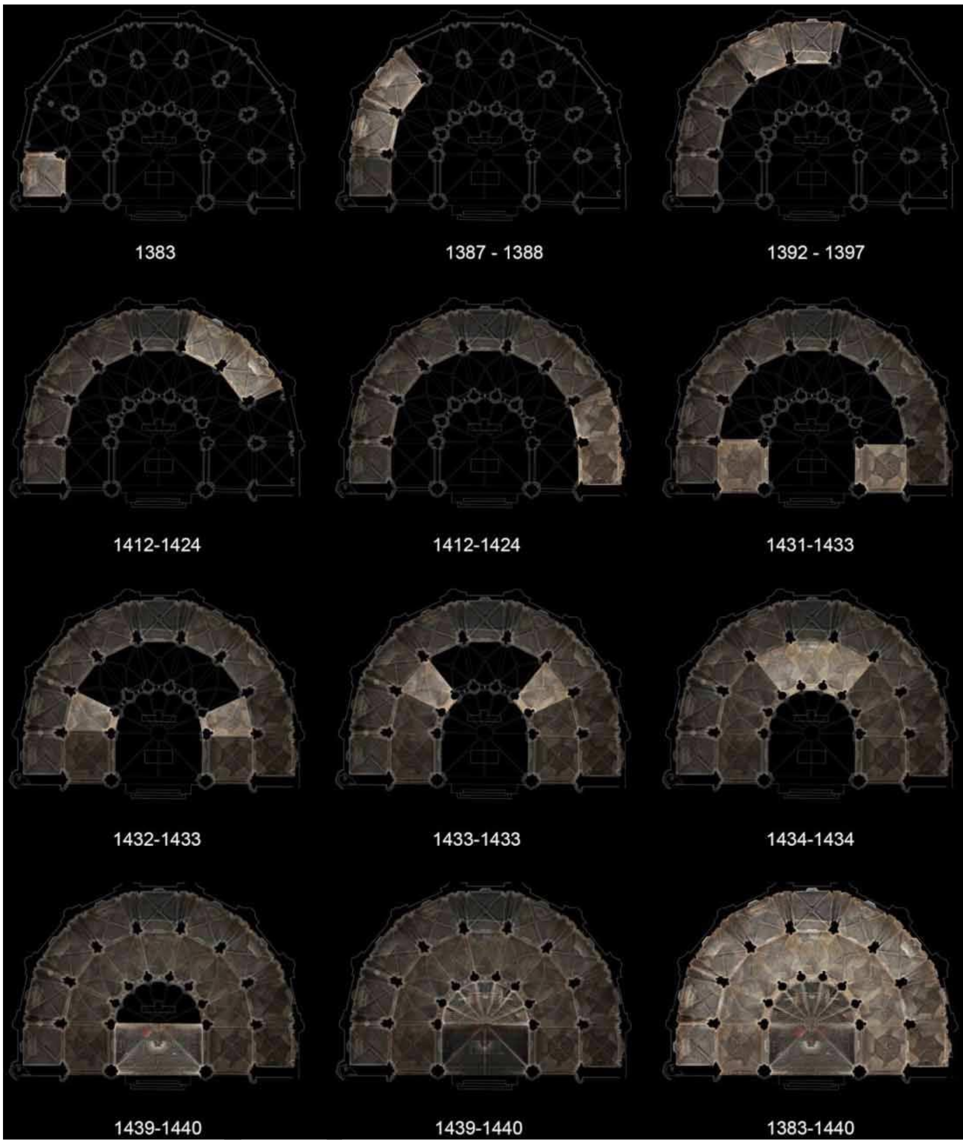

Figure 10. Revised chronology of the construction (J. Lluis, A. Costa, J. M. Puche, S. Coll, M. A. Soriano-Montagut, M. López and C. Soler, 2013)

The construction strategy in the case of the ambulatory (1432–1434) was different from the one used in the chapels, which were roofed consecutively. Roofing of the ambulatory thus began symmetrically, with the chapels of the gospel and epistle. Those with a square plan were done first, followed by the seven with trapezoidal plans.

The results show a pattern in the variation of heights that is the same as in the constructive process of the vaults shown. The end vaults thus have a larger difference in height between the keystone and the other points, while there is a progressive loss of height, which increases once again in the central vaults.

Deviations in the constructed model

The theoretical model of the cross-section is implemented according to proportional rules based on the layout of the floor. Although there are no documents with the original planimetry, by knowing these rules and the metric used it is possible to deduce its measures with precision. As stated previously, many factors can change the geometry of the masonry constructed, but the comparison between the theoretical model and the virtual 3D reconstruction provides relevant information about displacements suffered by the masonry.

Metric analysis of the 3D model reveals that in the vaults of the chapels, most of the keystones (taking as reference the height in its neck, Hb) are higher than its theoretical position. The most extreme distortions in absolute values are 19 cm higher in C2 (h =10.642 cm), and 18 cm lower in C8 (h = 10.270 cm). Also, five of the nine keystones are over its reference height of 10.45 cm, with an average deviation of 12 cm.

On the other hand, in the ambulatory all the vault keystones are lower than their theoretical height. The variations identified range from 8 cm in G9 (h = 16.652 cm) to 29 cm in G2 (h = 16.439 cm). The average deviation in this case is 23 cm lower. Finally, the height of the main keystone in the presbytery is variable, since the intersection between its neck and the severies does not happen in the same horizontal plane. Otherwise, most of the measures are almost coincident to the theoretical height of 2323 cm.

In relative terms, the deviations range between 0.5% and 1.8% compared to the theoretical plan. Although some absolute values may be relatively high, the geometric alterations of the masonry do not therefore exceed 2% in any case. If it is taken into account that part of these strains probably took place during the process of staking, construction and decentring, the structure can be said to have barely experienced any significant geometric alterations over the centuries.

Conclusions

The geometry of masonry constructions can be altered for different reasons, sometimes over several centuries. The deformations caused by the execution and centring of vaults (type 1), and by the decentring of vaults (type 2) are hardly distinguishable. However, both constructive deviations of the theoretical model are perfectly defined in the survey, and the geometry of the vaulting of the Gothic Cathedral of Tortosa can be seen to have changed substantially compared to what was predicted at the outset of the study.39

The methodology used enables the geometrical assessment of the vaults and the identification of formal deviations compared to a geometrical reference, in this case, the theoretical Gothic section. Thus, the geometry of the vaults can be parameterised through the 3D model, accurately locating the development of arches and severies (Figure 11).

The numerical treatment of this information brings to light some structural details about the configuration of the vaulting. The layout of the vaulted ceiling in the old style was known,40 the keystone is placed higher than the formerets and transverse arches, while these did not have the keystone at the same height.41 But the new data has enabled us to determine accurately the ratio between heights, as well as to quantify formal differences between vaults.

Moreover, the topographical variations enable the identification of geometric patterns that can be related to the execution process, which could be deduced from the historical sources and the study of the masonry. Thus, it was possible to observe a significant variation in the geomètric layout of vaults C8 and C9 compared to the others. A change in the construction criteria during the theoretical Mastership of Pasqual Xulbi is revealed that is not mentioned in the Llibres d'Obra (LLo.), which means that they could have been built under the Mastership of Joan Xulbi. So, the results of the methodology discussed here provide new data compatible with the information from direct sources.

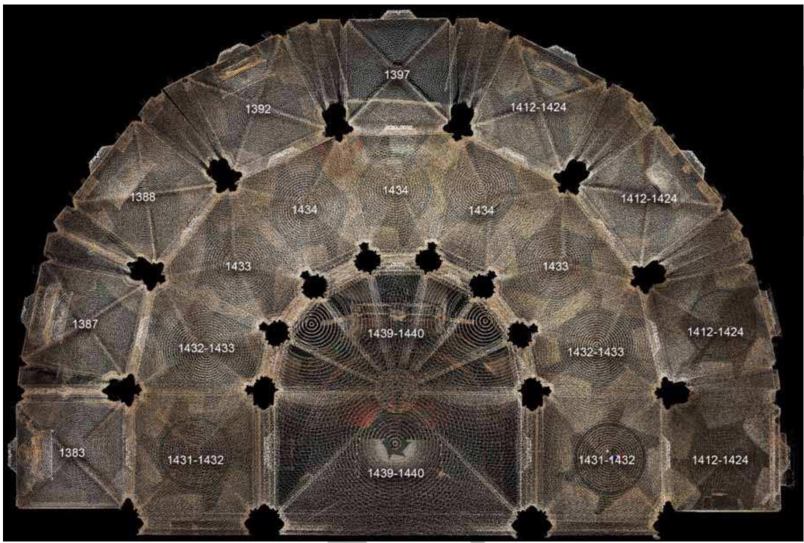

Figure 11. Aerial view of the point cloud (J. Lluis, A. Costa, J. M. Puche, S. Coll, M.A. Soriano-Montagut,

In construction terms, the canting in chapels C8 and C9 is important, since it exerts a passive pressure so that the entire ribbed vaulting acts on the pillar of the ambulatory in this manner. It will cause the deviation of the equilibrium conditions42 because of the asymmetry of the vault shape.

To summarise, the use of a Terrestrial Laser Scanner has led to improved knowledge about the Gothic Cathedral of Tortosa, providing new information about its history, construction and geometry.

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Abbreviations

TLS: Terrestrial Laser Scanner
Ll.o.: Llibres d'Obra (cathedral construction accounts),
ACTo: Arxiu Capitular de Tortosa (Tortosa Chapter Archive)
AHCTE: Arxiu Hitòric Comarcal Terres de l'Ebre (local Historical Archive)
FBMPM: Fundacion Bertomeu March Palma de Mallorca (Fundation Bertomeu March in Palma, Mallorca)

Notes

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